WORKBOOK

Subject: **Mathematics**

Student: ....

School year: ..../............

Topic: Polynomial

Polynomial is an expression constructed from one or more **variables and constants**, using the operations of addition, subtraction, multiplication, and raising.
Polynomials are one of the most important concepts in algebra and throughout mathematics and science.
Every polynomial in one variable is equivalent to a polynomial with the form:

$$a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + a_{n-2} \cdot x^{n-2} + \ldots\ldots + a_2 \cdot x^2 + a_1 \cdot x^1 + a_0$$

coefficients $a_1, \ldots\ldots a_n$ are real numbers

A polynomial can be written as the sum of one or more non-zero **terms....** $a_k \cdot x^k$, where $0 \le k \le n$

a_0 ... absolute term a_1 ... linear term a_2 ... quadratic term a_3 ... cubic term

For example $x^2 - 4x + 7$

The exponent on a term tells you the **"degree" of the term.**

Polynomials are also sometimes named for their degree:

- a second-degree polynomial, such as $4x^2$, $x^2 - 9$, or $ax^2 + bx + c$, is also called a "quadratic"
- a third-degree polynomial, such as $-6x^3$ or $x^3 - 27$, is also called a "cubic"
- a fourth-degree polynomial, such as x^4 or $2x^4 - 3x^2 + 9$, is sometimes called a "quartic"
- a fifth-degree polynomial, such as $2x^5$ or $x^5 - 4x^3 - x + 7$, is sometimes called a "quintic"

examples:
zero- polynomial

polynomial of degrese one (linear)

polynomial of degrese two (quadratic)

.....

polynomial of degrese 23

constant polynomial

inverse polynomial

Addition, subtraction, multiplicationand division of polynomials

Some formulas:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
$$a^2 - b^2 = (a-b)(a+b)$$
$$a^2 + b^2 \ldots nelze!!!$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Exercises:

1. Give the degree of the polynomial $-x^3 + 2x^2 - \pi x + 3$. Give the coefficients a_0, a_1, a_2, a_3. Write the quardatic term.

2. We have the polynomial $3x^4 - 2x^3 + 3$ Determine the degree and all its coefficients (a_n, a_{n-1},.....a_2, a_1, a_0), identify the value of the coefficient a_{n-2}.

3. Simplify the polynomials:

- $2x^2 + 3x - 4 - x^2 + x + 9$
- $10x^3 - 14x^2 + 3x - 4x^3 + 4x - 6$
- $25 - (x + 3 - x^2)$

Adding polynomials

$(2x + 5y) + (3x - 2y)$ *$5x + 3y$*

$(3x^3 + 3x^2 - 4x + 5) + (x^3 - 2x^2 + x - 4)$ *$4x^3 + 1x^2 - 3x + 1$*

$(7x^2 - x - 4) + (x^2 - 2x - 3) + (-2x^2 + 3x + 5)$ *$6x^2 - 2$*

$(x^3 + 5x^2 - 2x) + (x^3 + 3x - 6) + (-2x^2 + x - 2)$ *$2x^3 + 3x^2 + 2x - 8$*

Subtracting polynomials

$(x^3 + 3x^2 + 5x - 4) - (3x^3 - 8x^2 - 5x + 6)$ *$-2x^3 + 11x^2 + 10x - 10$*

$(6x^3 - 2x^2 + 8x) - (4x^3 - 11x + 10)$ *$2x^3 - 2x^2 + 19x - 10$*

Polynomial multiplication

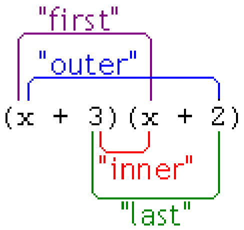
Simplify

a) $(5x^2)(-2x^3)$
b) $-3x(4x^2 - x + 10)$

The next step up is a two-term polynomial times a two-term polynomial.

Simplify $(x + 3)(x + 2)$

There is also a special method, useful ONLY for a two-term polynomial times another two-term polynomial. The method is called "FOIL". The letters F-O-I-L come from the words "first", "outer", "inner", "last", and are a memory device for helping you remember how to multiply horizontally, without having to write out the distribution like I did, and without dropping any terms. Here is what FOIL stands for:



So : $(x + 3)(x + 2) = x^2 + 2x + 3x + 6 = \mathbf{x^2 + 5x + 6}$

Simplify

a) $(x - 4)(x - 3)$
b) $(x - 3y)(x + y)$

Simplify

a) $(4x^2 - 4x - 7)(x + 3)$ [$4x^3 + 8x^2 - 19x - 21$]
b) $(x + 2)(x^3 + 3x^2 + 4x - 17)$ [$x^4 + 5x^3 + 10x^2 - 9x - 34$]
c) $(3x^2 - 9x + 5)(2x^2 + 4x - 7)$ [$6x^4 - 6x^3 - 47x^2 + 83x - 35$]
d) $(x^3 + 2x^2 + 4)(2x^3 + x + 1)$ [$2x^6 + 4x^5 + x^4 + 11x^3 + 2x^2 + 4x + 4$]

Polynomial Division

Divide

a) $x^2 - 9x - 10$ by $x + 1$

b) $3x^3 - 5x^2 + 10x - 3$ by $3x + 1$ $\qquad x^2 - 2x + 4 + \frac{-7}{3x+1}$

c) $2x^3 - 9x^2 + 15$ by $2x - 5$ $\qquad x^2 - 2x - 5 + \frac{-10}{2x-5}$

d) $4x^4 + 3x^3 + 2x + 1$ by $x^2 + x + 2$

$$4x^2 - x - 7 + \frac{11x + 15}{x^2 + x + 2}$$