WORKBOOK

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Subject: Mathematics

Student: .....

School year: ...../.....

Topic: Solving equations

Solving Absolute-Value Equations

When you take the absolute value of a number, you always end up with a positive number (or zero). Whether the input was positive or negative (or zero), the output is always positive (or zero).

For instance, | 3 | = 3, and | –3 | = 3

❖ **Solve | *x* | = 3**

| 3 | = 3 and | –3 | = 3, so *x* must be 3 or –3

- use the positive / negative property of the absolute value **to split the equation into two cases:**
  $x \geq 0$ *or* $x < 0$

- use the fact that the **minus sign " – " indicates "the opposite sign",** not necessarily a negative number.

Solving:

a) $x \geq 0$ **then |x| = x** ⟶ **|x|=3, x=3**

b) $x < 0$ **then |x| = -x** ⟶ **|x| = 3, - x = 3, x = -3**

Then the solution is ***x* = –3, 3**

❖ **Solve | *x* + 2 | = 7**

a) $x + 2 \geq 0$ **then |x + 2| = x +2** ⟶

| *x* + 2 | = 7
x + 2 = 7
x = 5

b) $x + 2 < 0$ **then |x + 2| = -(x +2)** ⟶

| *x* + 2 | = 7
–(*x* + 2) = 7¨
–*x* – 2 = 7
–9 = *x*

Then the solution is ***x* = –9, 5**.

Solve each of the following equations:

1) 0 = |x - 5| -2
2) |1 - 2x| = 5x + 10
3) 1 - |x - 3| = x – 2

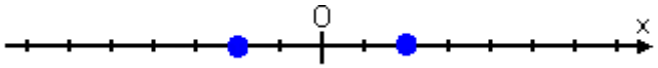
Use „the zero point method“ for solving equations with more than one absolute value

4) |x - 2| = 3 |x - 4|

5) $3 + |x| = 7 - |2 + x|$
6) $|x + 1| - 1 + |2 - x| - |x + 3| = 3$

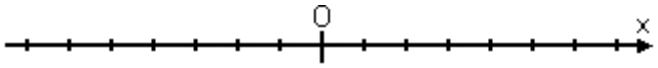
Absolute-Value Inequalities

Let's first return to the original definition of absolute value: "| *x* | is the distance of *x* from zero." For instance, since both –2 and 2 are two units from zero, we have | –2 | = | 2 | = 2:

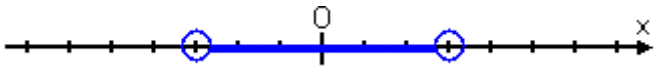


❖ **Solve | *x* | < 3.**

The solution is going to be all the points that are less than three units away from zero. Look at the number line:



The number 1 will work, as will –1; the number 2 will work, as will –2. But 4 will not work, and neither will –4, because they are too far away. Even 3 and –3 won't work (though they're right on the edge). But 2.99 will work, as will –2.99. In other words, all the points between –3 and 3, but not actually including –3 or 3, will work in this inequality.



❖ **Solve | 2*x* + 3 | < 6**

a) $2x + 3 \geq 0$ then $|2x + 3| = 2x + 3$ ⟶

$2x + 3 \geq 0$ and | 2*x* + 3 | < 6
$2x \geq -3$ 2*x* + 3 < 6
$x \geq -3/2$ *x* < $3/2$

$P_1 = \langle -3/2; 3/2)$
P

b) $2x + 3 < 0$ then $|2x + 3| = -(2x + 3)$ ⟶

$2x + 3 < 0$ and | 2*x* + 3 | < 6
$2x < -3$ -(2*x* + 3)< 6
$x < -3/2$ -2*x* – 3 < 6
x >- $9/2$

$P_2 = \langle -\frac{9}{2}; -\frac{3}{2})$

Then the solution to | 2*x* + 3 | < 6 is the interval P = $\langle -\frac{9}{2}; \frac{3}{2})$

❖ **Solve | 2*x* – 3 | > 5** [two intervals *x* < –1 and *x* > 4]

Note: Use „the zero point method“ for solving inequations with more than one absolute value

Solve each of the following inequations:

1) $|x - 1| + |x| > 4$
2) $|2x - 1| + |x - 2| \geq 1$
3) $|u + 3| \geq 8 - |2 - 3u|$
4) $|2x - 1| - |x - 2| \geq 1$

Uses:

www.purplemath.com

www.analyzemath.com

http://www.themathpage.com/alg/absolute-value.htm