



WORKBOOK

<http://agb.gymnaslo.cz>



Subject: Mathematics

Student:

School year:/.....

Introduction to Functions

In everyday life, many quantities depend on one or more changing variables. For example:

- a) Plant growth depends on sunlight and rainfall
- b) Speed depends on distance travelled and time taken

A function is a rule that relates how one quantity depends on other quantities

Example 1

We know the equation for the area, S , of a circle from primary school:

$S = \pi r^2$, where r is the radius of the circle

This is a **function** as each value of the independent variable r gives us **one** value of the dependent variable S .

Fill the table:

r	1	2	3	4	5	10	20
$S = \pi r^2$							



Example 2

It is given the set $M = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

x	2	3	4	5	6	7	8	9	10
y	0	1	0	0	1	0	0	1	0

Find the function:

Example 3

It is given the prism

Find a **function** as each value of the independent variable a gives us **one** value of the dependent variable u

a.. the side

u... the space diagonal

- We use x for the independent variable and y for the dependent variable for general cases.
- In general, the value of any function $f(x)$ when $x = a$ is written as $f(a)$.

Example 4

We have $f(x) = \frac{5-x}{x+2} \quad x \in (-1; 5)$

Find $f(-1)$ $f(0,6)$ $f(2)$ $f(4,2)$

- Function Notation

We normally write functions as: $f(x)$ and read this as "function f of x ".

We can use other letters for functions, like $g(x)$ or $y(x)$.

Repetition:

1. Given $f(x) = 3x + 20$, find $f(-4)$ $f(10)$

2. Given that the height of a particular object at time t is

$h(t) = 50t - 4.9t^2$, find $h(2)$ $h(5)$

3. If $F(t) = 3t - t^2$ for $t \leq 2$, find $F(2)$ and $F(3)$.

The Graph of a Function

The graph of a function is the set of all points whose co-ordinates (x, y) satisfy the function $y = f(x)$. This means that for each x -value there is a corresponding y -value which is obtained when we substitute into the expression for $f(x)$.

Since there is no limit to the possible number of points for the graph of the function, we will follow this procedure at first:

1. Select a few values of x (at least 5)
2. Obtain the corresponding values of the function and enter them into a table
3. Plot these points by joining them with a smooth curve

Example 1

Graph the function $y = 2x + 1 \quad x \in \mathbb{R}$

Domain and Range of a Function

Domain $D(f)$

The domain of a function is the complete set of possible values of the independent variable in the function.

In plain English, this definition means:

The **domain** of a function is the set of all possible x values which will make the function "work" and will output real y -values.

When finding the **domain**, remember:

- The denominator (bottom) of a fraction **cannot be zero**
- The values under a square root sign **must be positive**

Range $H(f)$

The **range** of a function is the complete set of all possible **resulting values** of the dependent variable of a function, after we have substituted the values in the domain.

In plain English, the definition means:

The **range** of a function is the possible y values of a function that result when we substitute all the possible x -values into the function.



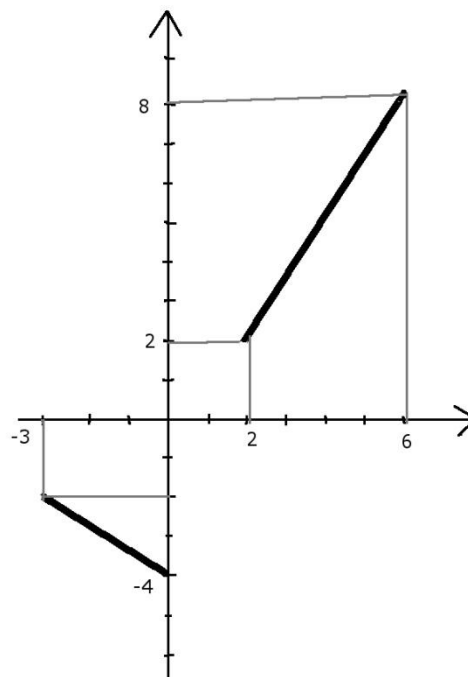
When finding the **range**, remember:

- Substitute different **x**-values into the expression for **y** to see what is happening
- Make sure you look for **minimum** and **maximum** values of **y**
- **Draw a sketch!** In math, it's very true that a picture is worth a thousand words.

Examples

1. Look at the picture:

- write $D(f)$
- $f(0) =$ $f(4) =$ $f(-3) =$
- write $H(f)$
- find $x \in D(f)$, $f(x) = -2$, $x =$
- $f(x) = 8$, $x =$



2. Find the domain and range for the function

$$f(t) = \frac{1}{t+2}$$

3. Find the domain and range for the function

$$g(s) = \sqrt{3-s}$$

4. Find the domain and range of the function defined by the coordinates:

$$h(x) = \{[-4, 1], [-2, 2.5], [2, -1], [3, 2]\}$$

Source: <http://www.intmath.com/>

