Gymnázium, Brno, Slovanské nám. 7

WORKBOOK

http://agb.gymnaslo.cz

Subject: **Mathematics**

Student: ....

School year: ..../...........

Topic: **Trigonometry**

Trigonometrical ratios in a right-angled triangle

Tangent ratio

Study the diagram. The side of the triangle are named in relation to angle A. The longest side AB is called the ***hypotenuse***, the side next to the angle is called the ***adjacent*** side, and the third side BC is ***opposite*** angle A.

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INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

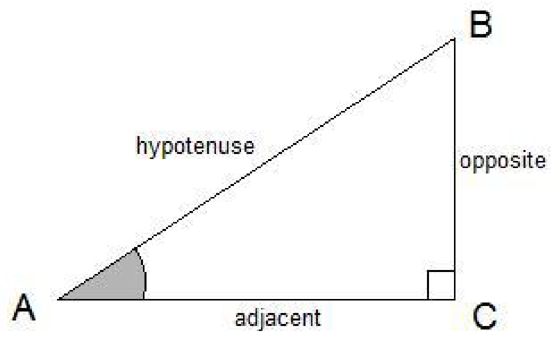
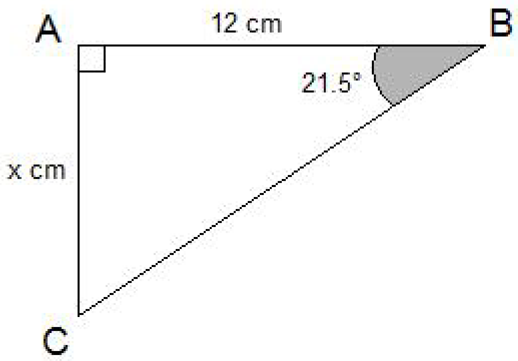


Figure 1.

If you find the ratio $\frac{BC}{AC}$, you will find it always has the same value however large you draw the triangle, provided angle A remains the same. We have a name for this ratio. We call it the ***tangent*** of the angle.

tangent of A = $\frac{BC}{AC} = \frac{adjacent}{opposite}$ or tan A = $\frac{BC}{AC} = \frac{adjacent}{opposite}$

Example:
Find the value of *x*



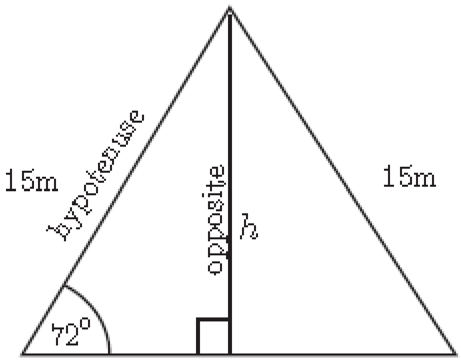
[*x* = 4.73 cm]

Sine ratio

Referring back to *Figure* 1. If you find the ratio $\frac{AB}{BC}$, you will find it also has a constant value provided the angle A remains unaltered. This ratio is called the ***sine*** of angle A or sin A.

sin A = A = $\frac{AB}{BC} = \frac{opposite}{hypotenuse}$

Example:
Suppose we wish to find the height of the isosceles triangle. In this example we want to know the length, h say, of the side opposite the angle of 72°, and we know the length of the hypotenuse. The ratio which links the opposite and the hypotenuse is the sine.

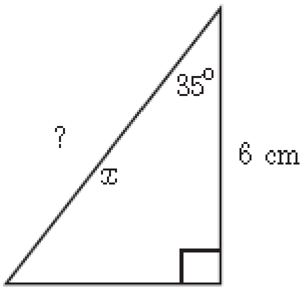


[14,3 m]

Cosine ratio
Once again, referring back to *Figure* 1, if you find the ratio $\frac{AC}{AB}$, you will find it has a constant value provided the angle A remains constant. This ratio is called the ***cosine*** of angle A or cos A

cos A = $\frac{AC}{AB} = \frac{adjacent}{hypotenuse}$

Example:
Consider the right-angled triangle shown in the picture. Suppose we wish to find the length of the hypotenuse.



[7.32 m]

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} \qquad \sin A = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Exercises:

1) The angle of elevation of the top of a tree from a point on the ground 10m from the base of the tree is 28°. What is the height of the tree (to 1 decimal place)?

2) Using a surveying instrument 1.6m high, the angle of elevation of the top of a tower is measured to be 55° from a point 6m from the base of the tower. How high is the tower (to 1 decimal place)?

3) The angle of elevation of the top of a 20m high mast from a point at ground level is 34°. How far is the point from the foot of the mast (to 1 decimal place)?

4) An isosceles triangle has base 8 cm and sloping sides both with length 10 cm. What is the base angle of this triangle (to the nearest degree)?

5) A supporting cable of length 30m is fastened to the top of a 20m high mast. What angle does the cable make with the ground? How far away from the foot of the mast is it anchored to the ground (to 1 decimal place)?

6) A right-angled triangle has sides 5, 12, 13. What is the size of the smallest angle in this triangle (to the nearest degree)?
7) What is the height (to 1 decimal place) of an isosceles triangle with base angle 65° and sloping sides with length 10 cm? What is the length of the base of this triangle (to 1 decimal place)?

8) One angle in a right angled triangle is 50° and the side opposite this angle has length 5 cm. What is the length of the hypotenuse (to 1 decimal place)?

9) In a right angled triangle, the hypotenuse has length 8m and one angle is 55°. What is the length of the shortest side (to 1 decimal place)?

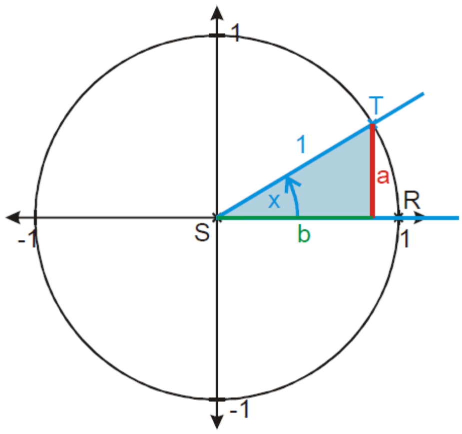
Answers:

1) 5.3 m **2)** 10.2 m **3)** 29.7 m **4)** 66° **5)** 42°, 22.4 m **6)** 23° **7)** 9.1 cm, 8.5 cm **8)** 6.5 cm **9)** 4.6 m

Trigonometric ratios of an angle of any size

The sine function f(x) = sin x

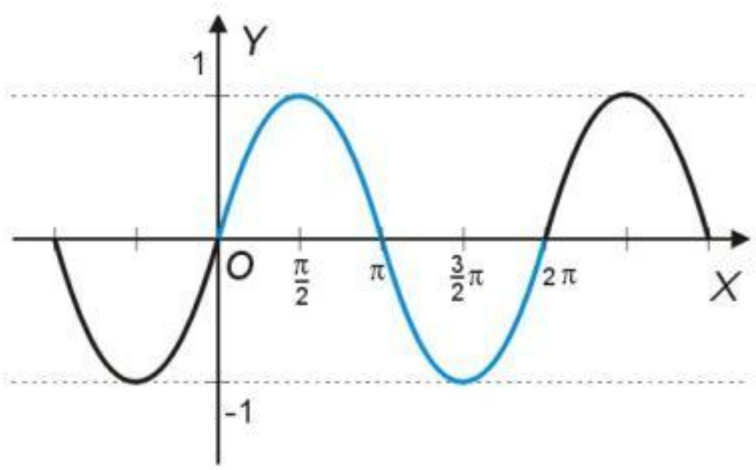
We shall start with the sine function $f(x) = \sin x$. This function can be defined for any number x using a diagram like this.



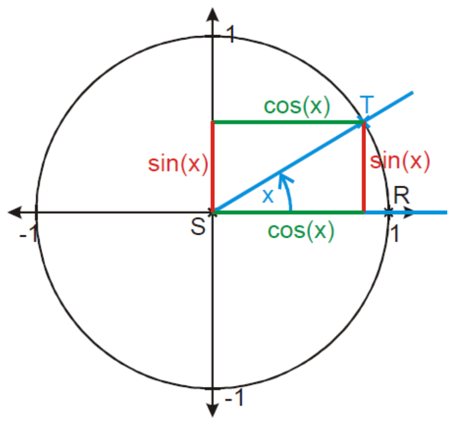
The information from this Figure can also be used to see how changing x affects the value of sin x. We can use a table of values to plot selected points between $x = 0°$ and $x = 360°$, and draw a smooth curve between them. We can then extend the graph to the right and to the left, because we know that the graph repeats itself.

x	0 °	30 °	45 °	60 °	90 °	225°	270°	315°	360°
Sin x									

We can use a table of values to plot selected points between $x = 0°$ and $x = 360°$, and draw a smooth curve between them. We can then extend the graph to the right and to the left, because we know that the graph repeats itself.



The cosine function f(x) = cos x



We shall now look at the cosine function, $f(x) = \cos x$. This function can be defined for any number x using a diagram like this.

We take a circle diagram similar to the one we used for the sine function. But now we look at the horizontal axis coordinate of the point where the line and the circle meet, to find the value of cos x.
The information from this picture can also be used to see how changing x affects the value of cos x. We can use a table of values to plot selected points between $x = 0°$ and $x = 360°$, and draw a smooth curve between them. We can then extend the graph to the right and to the left, because we know that the graph repeats itself.

x	0 °	30 °	45 °	60 °	90 °	225°	270°	315°	360°
cos x									

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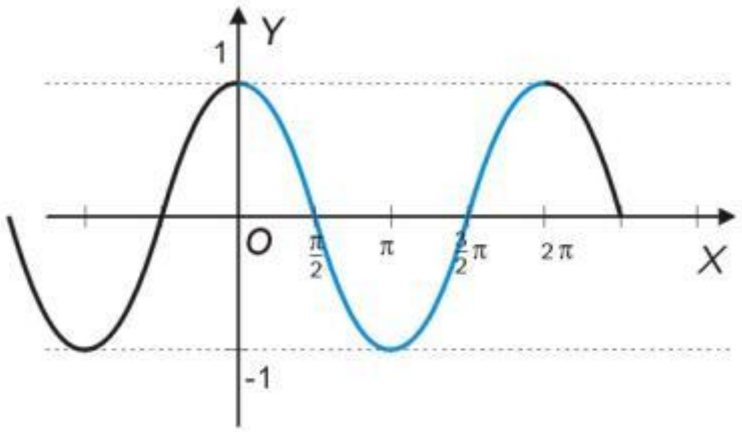
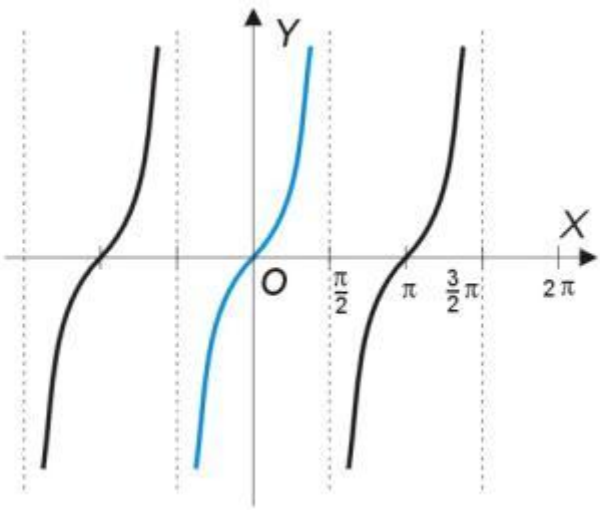
The information from this picture can also be used to see how changing *x* affects the value of cos *x*. We can use a table of values to plot selected points between *x* = 0° and *x* = 360°, and draw a smooth curve between them. We can then extend the graph to the right and to the left, because we know that the graph repeats itself.





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