



WORKBOOK

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Subject: Mathematics

Teacher:

Student:

School year:/.....

Conic section

The conic sections are the nondegenerate curves generated by the intersections of a plane with one or two nappes of a cone.

For a plane perpendicular to the axis of the cone, a **circle** is produced.

For a plane that is not perpendicular to the axis and that intersects only a single nappe, the curve produced is either an **ellipse** or a **parabola**.

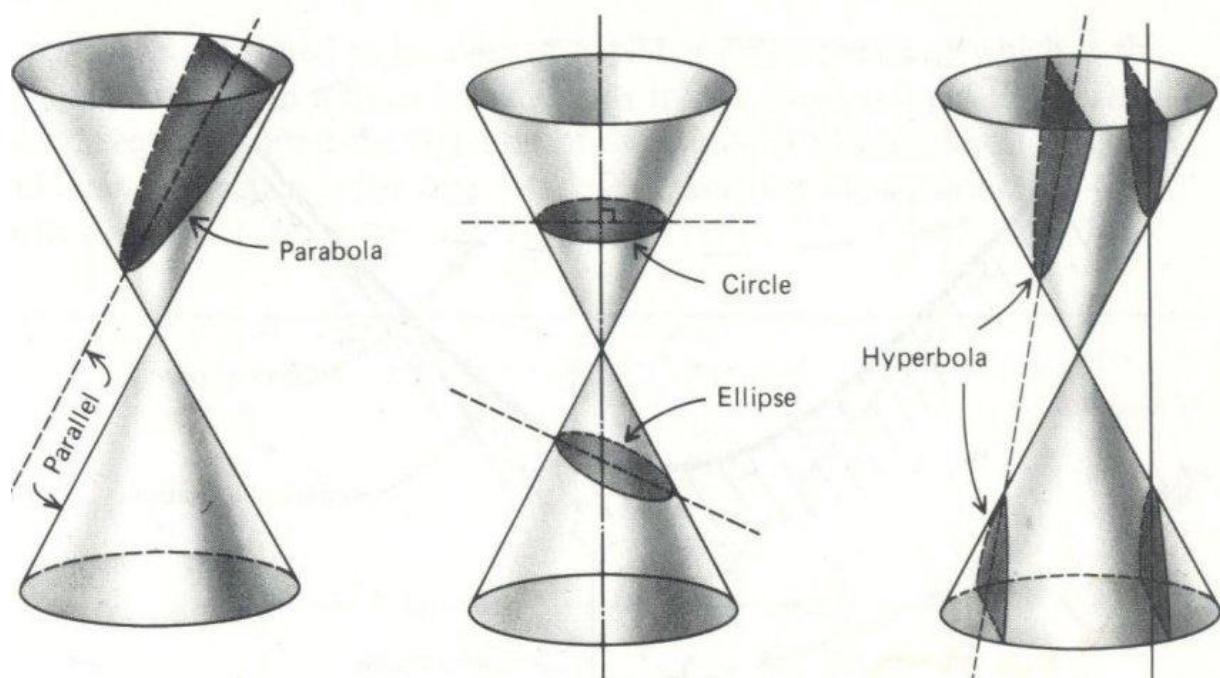
The curve produced by a plane intersecting both nappes is a **hyperbola**.

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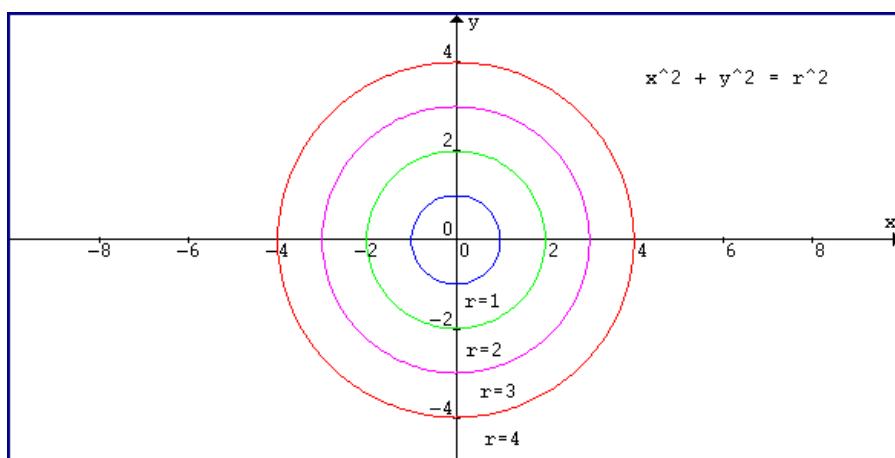
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Circle

A circle is a set of all points equidistant from a point.

The equation of a circle, centred at the origin, is $x^2 + y^2 = a^2$, where a is the radius.



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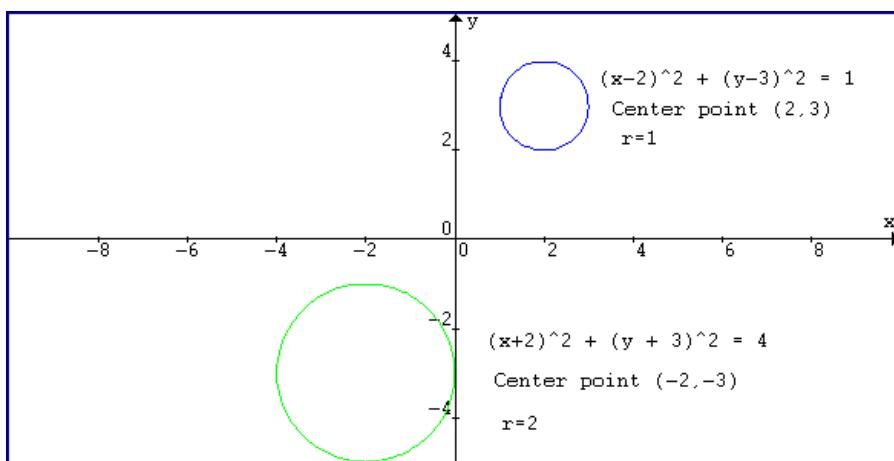
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A simple translation of the circle equation becomes:

$$(x - h)^2 + (y - k)^2 = r^2$$

With center at (h, k) and radius r .

Here are some examples:



Example:

- Identify the coordinates of the center and the length of the radius in the circle below

$$(X - 5)^2 + (y + 2)^2 = 4$$
- Graph a circle centered at (5,1) with a radius of 5.
- Find the equation of the circle with center (2,3) and radius 5.
- Find the center, radius and graph the equation: $(x - 2)^2 + (y + 5)^2 = 17$
- $x^2 + y^2 - 8x + 4y - 8 = 0$ Find center, radius and graph.
- Find the intersection of the line $y = x - 1$ and the circle $x^2 + y^2 = 25$.

Ellipse

If $F_1(c, 0)$ and $F_2(-c, 0)$ are two fixed points in the plane and a is a constant, $0 < c < a$, then the set of all points P in the plane such that

$$PF_1 + PF_2 = 2a$$

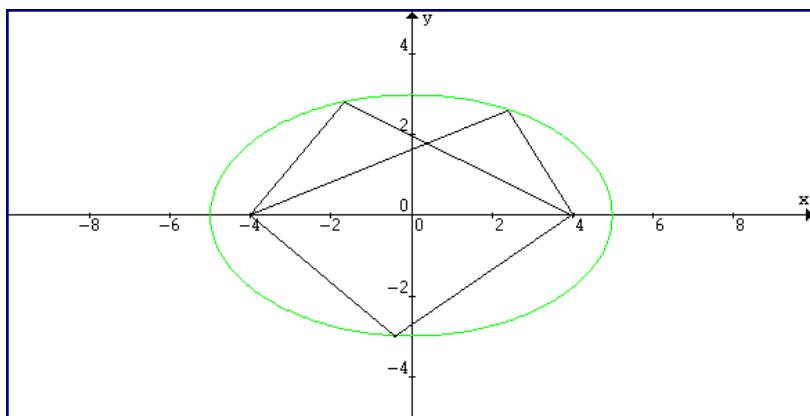
is an ellipse. F_1 and F_2 are the foci of the ellipse.

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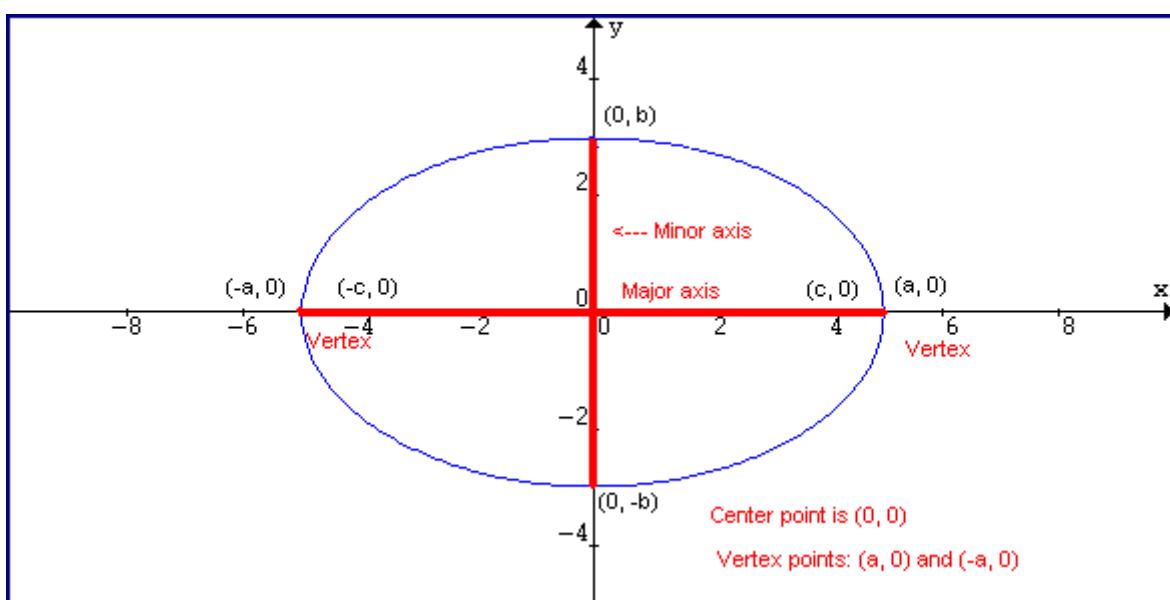


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equation of the ellipse:



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ with } b^2 = a^2 - c^2$$

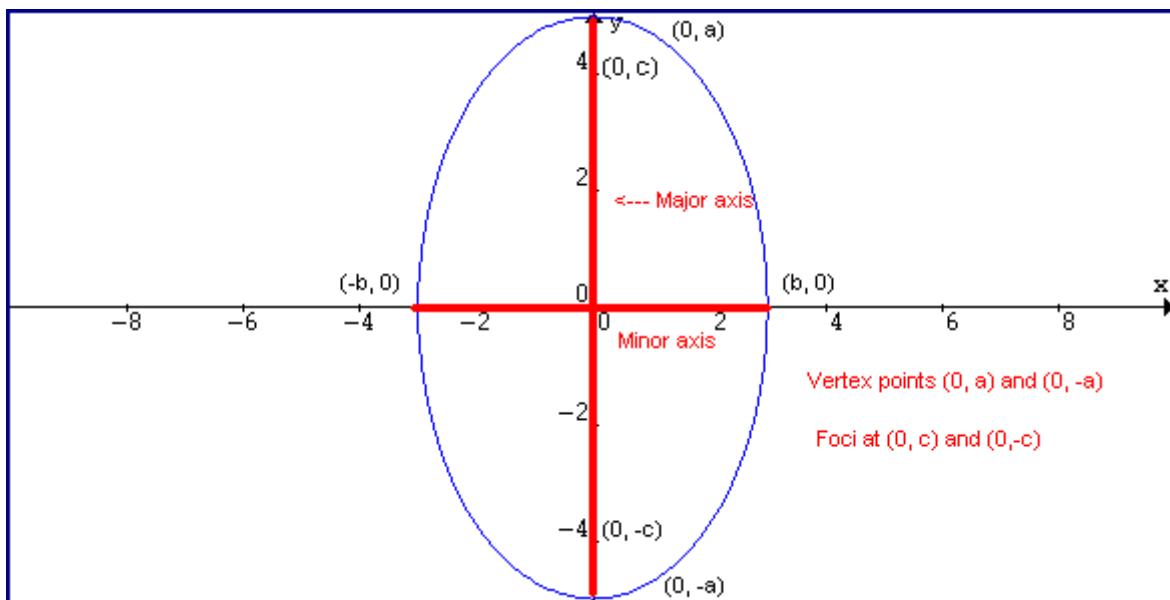
This ellipse has the major axis parallel to the x-axis making it open longer across. The length is $2a$. The minor axis is parallel to the y-axis and has length $2b$. The foci points are $2c$ units apart. The center of the ellipse until we translate it will remain at $(0, 0)$. The vertex points are at the end points of the major axis. Look at the equation. The a value is always the biggest number!!

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- Sketch the graph and find the vertices, end points of the minor axis and foci points for:
 $x^2 + 4y^2 = 16$
- An ellipse has its center at the origin. Find an equation of the ellipse with Vertex $(8, 0)$ and minor axis 4 units long.
- An ellipse has its center at the origin. Find an equation of the ellipse with vertex $(0, -12)$ and focus $(0, -4)$.

Translation of the Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

The center is now at (h, k) .

All values are now calculated from this point rather than from $(0, 0)$

- Sketch the graph and find the vertices, end points of the minor axis and foci for:

$$\frac{(x-2)^2}{25} + \frac{(y+1)^2}{16} = 1$$
- Find the equation of the ellipse with center $(2,5)$, one focus $(5,5)$ and one vertex $(7, 5)$.

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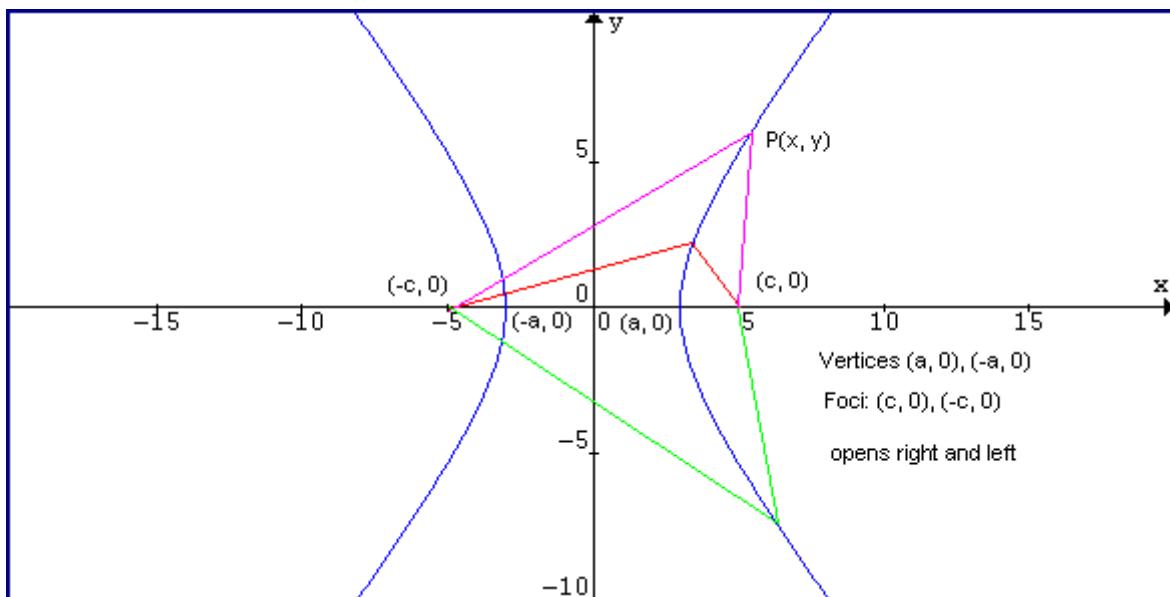
Hyperbolas

Definition of the hyperbola:

A hyperbola is the set of all points $P(x, y)$ in the plane such that $|PF_1 - PF_2| = 2a$

Again F_1 and F_2 are focus points. This time the difference of these distances remain a constant at $2a$.

The explanation is similar to that of the ellipse. Since the ellipse is the sum of the distances and the hyperbola is the difference of the distances, the equations are very similar. They differ only in the sign and the longest side for a hyperbola is c .



The equation of the above hyperbola would have the form:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \text{ where } c^2 = a^2 + b^2$$

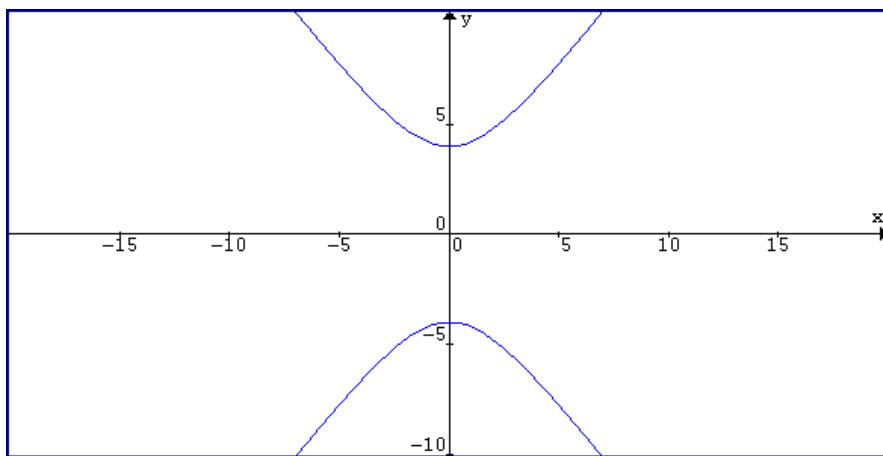
The hyperbola opens left and right. Notice it comes in two parts. Different than an ellipse which is a closed figure. Hyperbolas can also open up and down.

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This hyperbola has the form:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

To get the correct shape of the hyperbola, we need to find the **asymptotes** of the hyperbola.

The asymptotes are lines that are approached but not touched or crossed. These asymptotes are boundaries of the hyperbola. This is one difference between a hyperbola and a parabola. For the hyperbolas that open right/left, the asymptotes are:

$$y = \pm \frac{b}{a} x$$

and for hyperbolas opening up/down, the asymptotes are:

$$y = \pm \frac{a}{b} x$$

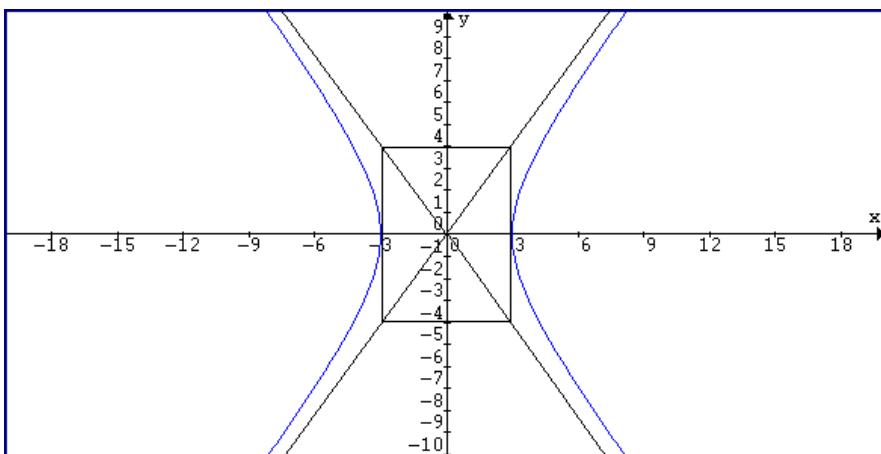
To form the asymptotes easily on the graph, all we need do is form a rectangle using a and b.

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Problems:

- Graph the hyperbola $\frac{x^2}{16} - \frac{y^2}{4} = 1$. Find the vertices, foci and equations of the asymptotic lines.
- Graph the hyperbola $\frac{x^2}{25} - \frac{y^2}{9} = 1$. Give the vertices, foci and equations of asymptotic lines.
- Find an equation of a hyperbola with center at the origin, one vertex at (7, 0) and a focus at (12, 0).

Translations of Hyperbolas

If the hyperbola opens right/left the translation is:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

with the equations of the asymptotic lines as:

$$y - k = \pm (b/a)(x - h)$$

If the hyperbola opens up/down the translation is:

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

with the equations of the asymptotic lines as:

$$y - k = \pm (a/b)(x - h)$$

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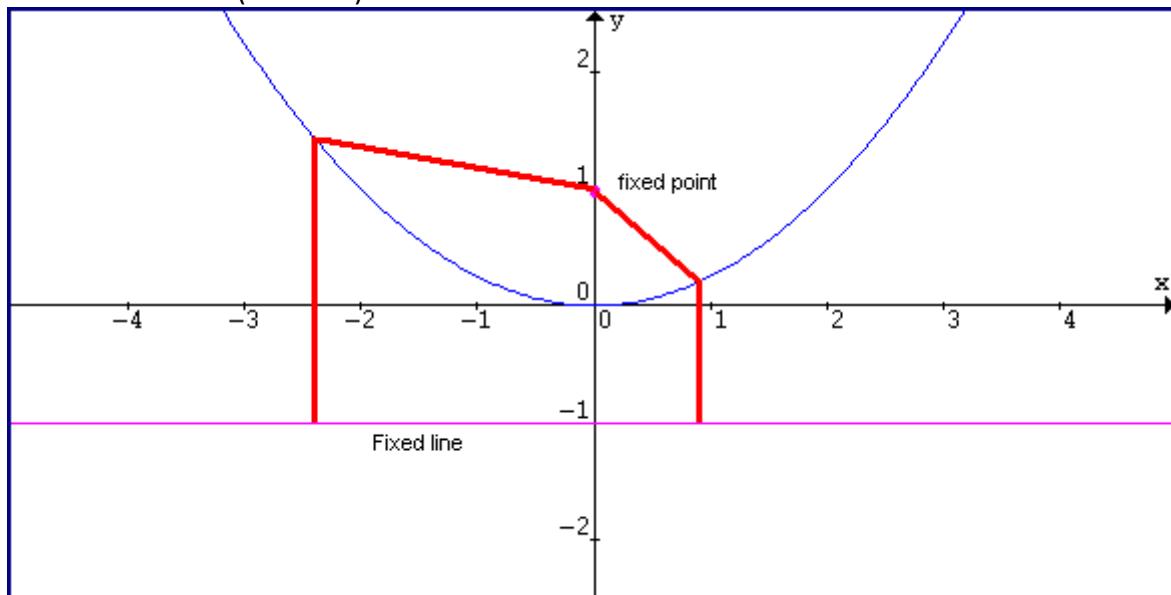
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Sample Problems:

- Graph the equation: $\frac{(y-2)^2}{36} - \frac{(x-1)^2}{25} = 1$. Find the center, vertices, foci and the equations of the asymptotic lines.
- Find the equation of a hyperbola with center (1, 1), vertex (3, 1) and focus at (5, 1).

Parabolas

parabola is the set of all points P in the plane that are equidistant from a fixed point F (focus) and a fixed line d (directrix).



The equations of the parabola are as follows:

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$$y = \frac{1}{4p}x^2 \quad \text{parabola opens up.}$$

$$y = \frac{-1}{4p}x^2 \quad \text{parabola opens down.}$$

$$x = \frac{1}{4p}y^2 \quad \text{parabola opens right.}$$

$$x = \frac{-1}{4p}y^2 \quad \text{parabola opens left}$$

For parabolas opening up/down, the directrix is a horizontal line in the form $y = \pm p$

For parabolas opening right/left, the directrix is a vertical line in the form

$$x = \pm p$$

The vertex point for all of the above is $(0, 0)$

Sample Problems

1. Find the focus point and directrix and graph the parabola: $y = x^2/8$
2. Find the focus point and the directrix and graph the parabola: $x = -2y^2$
3. Find the equation of the parabola with vertex at $(0, 0)$ and directrix $y = 2$.
4. Find the equation of a parabola with focus at $(2, 0)$ and directrix at $x = -2$

Translations of the parabola

The equations of the parabola with vertex (h, k) are:

$$y - k = \frac{\pm 1}{4p}(x - h)^2 \quad \text{opens up / down}$$

$$x - h = \frac{\pm 1}{4p}(y - k)^2 \quad \text{opens right / left}$$

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Sample Problems

1. Find the vertex, focus and directrix and graph the parabola $y = 2x^2 - 8x + 1$
2. Find the equation of the parabola with focus (1, 3) and directrix $x = -3$.

Uses: <http://home.windstream.net>

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